Introduction to Cosmology

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Plan

1. Lecture 1 : Qualitative description of the universe
2. Lecture 2 : Distances in the universe
3. Lecture 3 : The history of the Big Bang

Lecture 1 : Qualitative description of the universe

Plan

1. The universe before 1920
2. The 1920 revolutions
3. The standard universe in modern cosmology
4. General relativity
5. The Friedmann-Lemaître metric in flat space
6. The cosmological redshift

1. The universe before 1920

The universe is eternal (since its creation), static, and unchanging

Knowledge limited to our galaxy, the Milky Way : 100 000 light years (l-y). Sun about 28 000 l-y from center.

Some hints that Andromeda and Magellan clouds do not belong to Milky Way

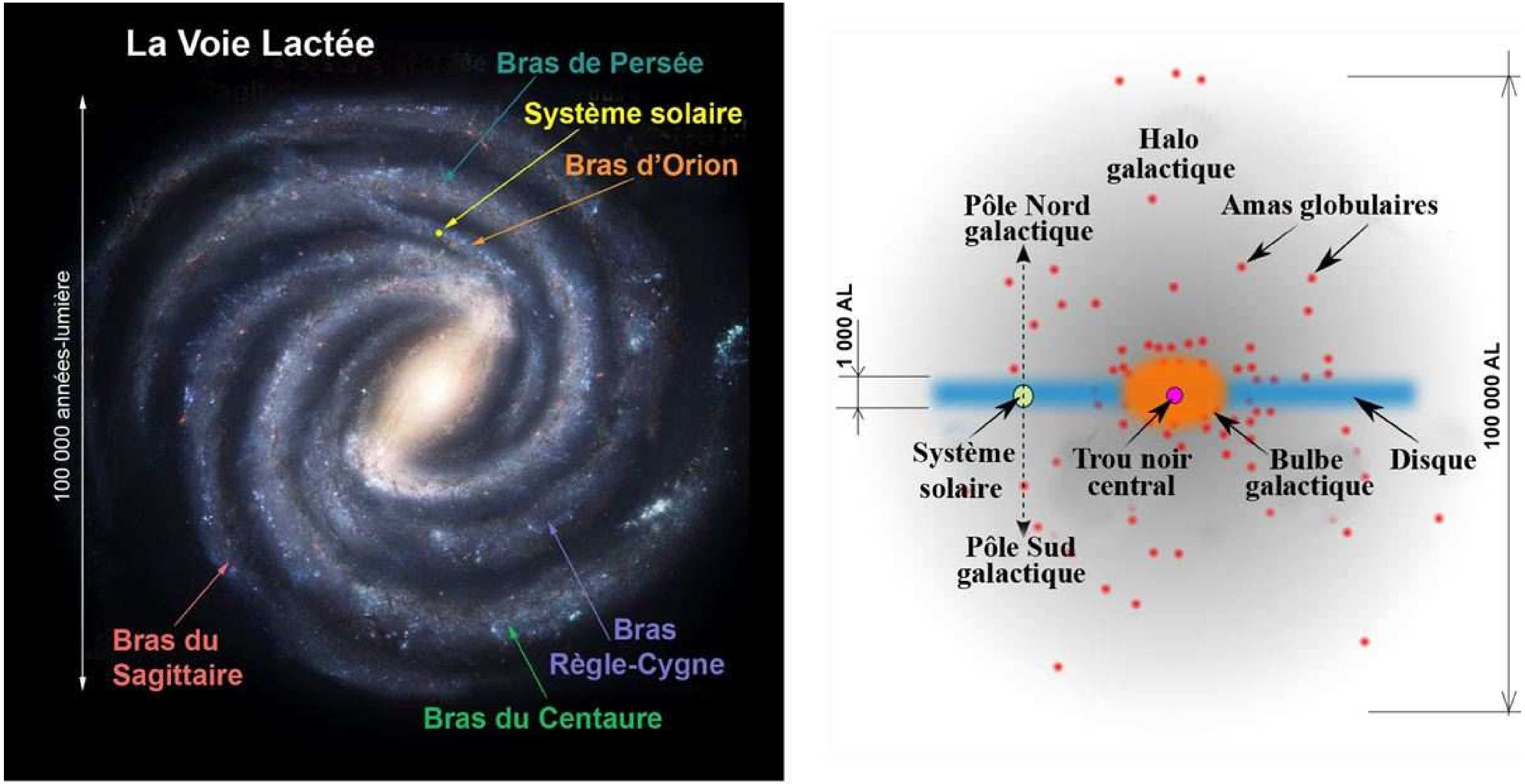

FIG. 1 – The Milky Way

FIG. 2 – Andromeda, the M31 galaxy. Spins too fast, stars should be ejected, spherical dark matter halo

Olbers paradox : at night, the sky should appear uniformly bright, any line of sight should meet a star. Analogy with a forest : in a dense forest, each line of sight meets a tree

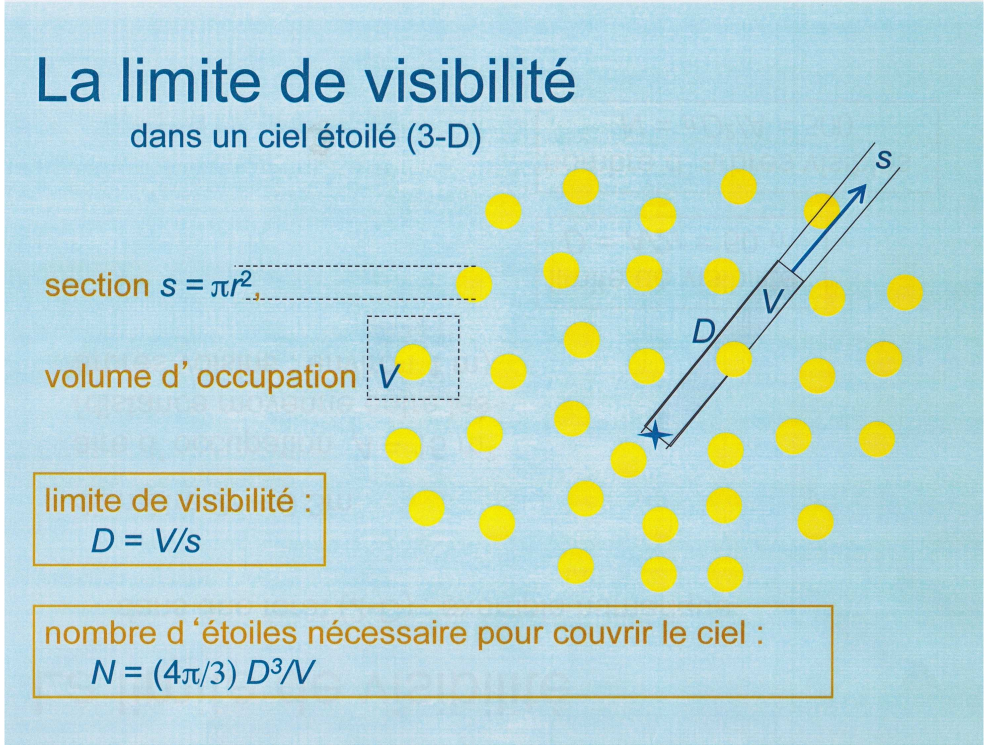

FIG. 3 – Distance of visibility in a static universe.

Visibility limit : $\sim 10^{22}$ l-y, while age of the universe + finite speed of light $\Longrightarrow$ we can see at most as far as $\sim 10^{10}$ l-y. Furthermore redshift, but marginal influence only.

However in microwave domain wee see a uniformly bright sky in all directions! Cosmic Microwave Background, or CMB : snapshot of the universe 13 billion years ago

2. The 1920 revolutions

FIG. 4 – The founding fathers of modern Cosmology. Albert Einstein (1879-1955) ; Edwin Hubble (1889-1953) ; Georges Lemaître (1894-1966).

Georges Lemaître : “As far as I can see, such a theory (e.g. the Big Bang) remains entirely outside any metaphysical or religious question.”

First revolution : Hubble

1 There are galaxies outside our Milky Way

2 The galaxies fly away from us with a speed proportional to their distance (flight of galaxies)

Second revolution : Einstein (General Relativity), Friedmann, Lemaître

3 Static universe incompatible with General Relativity : unstable solution

4 Expansion of the universe described by General Relativity : Friedmann-Lemaître metric

How do we measure distances? We know (or we assume to know) the luminosity of "standard candles"

1. Cepheids : relation period-luminosity
2. Type Ia supernovae, white dwarfs which accrete material from a neighboring star until they explode, peak luminosity believed to be known

Energy flux f function of the luminosity L and distance d

$$f = \frac{L}{4\pi d^2}$$

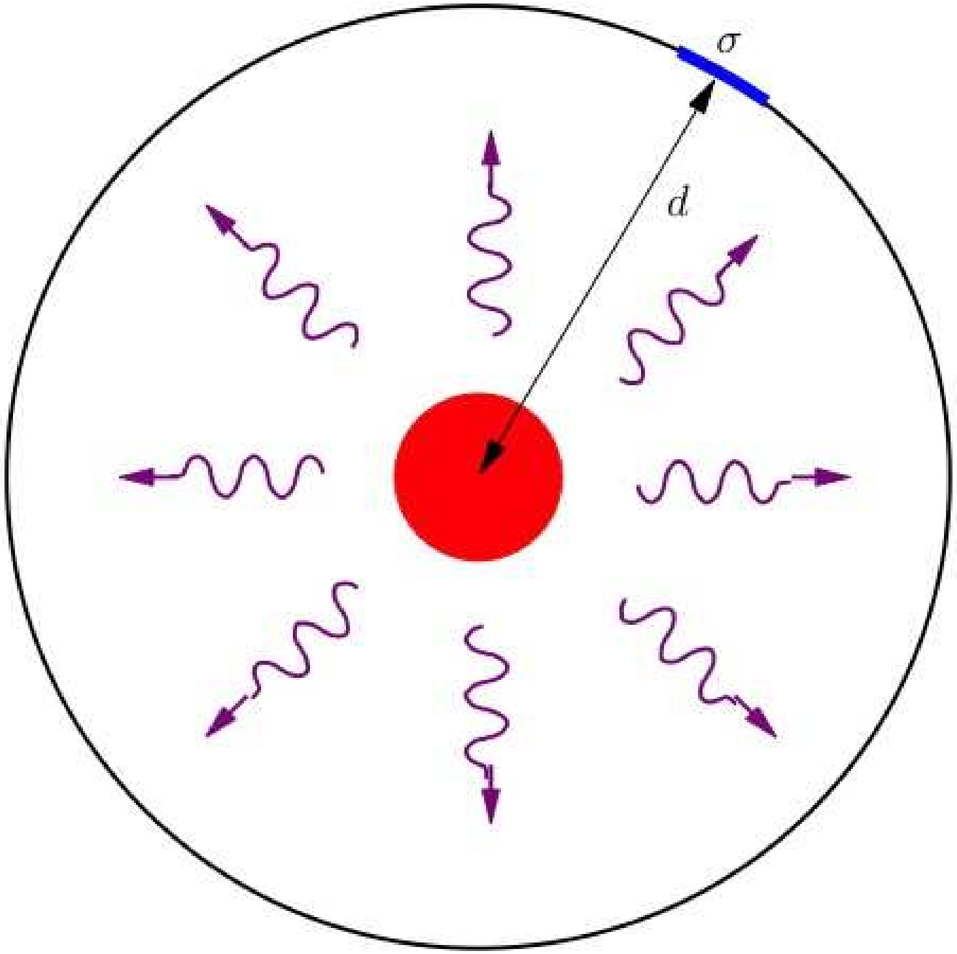

FIG. 5 – The flux-luminosity relation. $E = \sigma f = \sigma L/(4\pi d^2)$

3. The standard universe in cosmology

1. Over distances of about 500 millions l-y, the universe is homogeneous (has the same properties at each point) and isotropic (has the same properties in all directions). Of course local fluctuations : stars, galaxies, clusters of galaxies. Best evidence : isotropy of the Cosmic Microwave Background (CMB)

2. The matter we can observe, that which emits light or electromagnetic radiation, must be supplemented by dark matter. Dark matter does not interact with light and with ordinary matter, except for gravitational interactions, expect (G = gravitational constant)

$$\frac{GM(r)}{r^2} = \frac{v^2(r)}{r}$$

We do not find $v(r) \propto r^{-1/2}$ but rather $v(r)$ ∼constant. Other evidence : gravitational lensing.

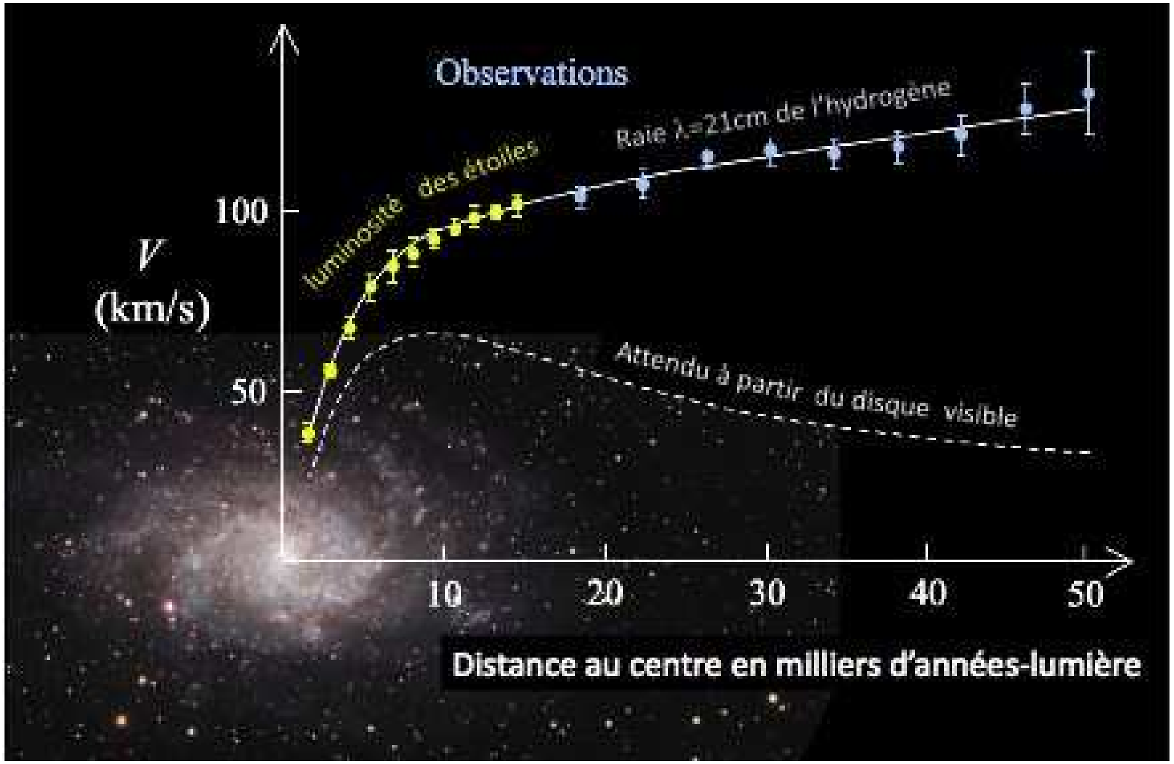

Fig. 6 – Angular velocity of stars and gas clouds in the Triangle Galaxy M33 as a function of distance from center.

3. Hubble's law, $v = H_0 d$, v = galaxy velocity, d = distance,
H_0 = Hubble's constant
Age t_0 of the universe, with constant expansion

$$v = H_0 d = H_0 v t_0 \qquad t_0 \simeq t_H = \frac{1}{H_0}$$

Latest values from Planck (2014) 1 parsec = 3.26 l-y

$$\begin{aligned} H_0 &= 67.3 \pm 1,2\,\mathrm{km.s^{-1}/Mpc} = 2.08 \pm 0,04 \times 10^{-18}\,\mathrm{s}^{-1}\,, \\ t_H &= \frac{1}{H_0} = 4.80 \times 10^{17}\,\mathrm{s} = 15.2 \times 10^9\,\mathrm{years} \end{aligned}$$

Cosmological redshift from Doppler effect $v/c \ll 1$

$$\frac{v}{c} = \frac{\Delta\lambda}{\lambda} \simeq z$$

4. General Relativity

Or better Einstein's theory of gravitation. Two ingredients

1. Universality of free fall : in a gravitational field, all objects fall at the same rate.
2. Equivalence principle : gravitational field locally equivalent to acceleration. Einstein's happiest thought

Metric. For a time-like interval gives the proper time τ : time as measured by a clock linked to an observer or a particle. Coordinates x^μ of a space-time point x, metric tensor $g_{\mu\nu}(x)$ $\mu, \nu = 0, 1, 2, 3$

$$\mathrm{d}\tau^2 = g_{\mu\nu}(x)\,\mathrm{d}x^\mu \mathrm{d}x^\nu$$

For a finite space-time interval

$$\tau(A \to B) = \int_A^B \sqrt{g_{\mu\nu}(x)\,\mathrm{d}x^\mu \mathrm{d}x^\nu}$$

In the absence of forces other than gravitation, a particle follows a space-time geodesic which maximizes its proper time.

Einstein equations. From the metric, compute the Ricci tensor $R_{\mu\nu}$, the scalar curvature $R = R^\mu{}_\mu$. Then

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = -8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$$

$T_{\mu\nu}$ = energy-momentum tensor, G = gravitational constant, Λ = cosmological constant. Einstein's theory is a local theory, does not care about global topology, for example plane $\equiv$ cylinder or torus

Cosmological constant : Einstein "biggest blunder of my life !"

Einstein equations determines geometry ($R_{\mu\nu}$), whence the metric ($g_{\mu\nu}$), as a function of energy ($T_{\mu\nu}$). Particle in free fall follows geodesics of the metric.

1. Matter tells space how to curve
2. Curvature tells matter how to move

5. The Friedmann-Lemaître metric

Because of isotropy, the three dimensional-space orthogonal to geodesics at a given time may be of three types only

1. A three dimensional sphere in a four dimensional-space (constant positive curvature)
2. A flat ordinary three dimensional space (zero curvature)
3. A three dimensional hyperbolic surface (constant negative curvature)

Fortunately, Nature has been kind, observation favors scenario (2). Galaxy (X, Y, Z), Euclidean distance from our own Galaxy, Pythagora's theorem

$$d(t) = [X^2 + Y^2 + Z^2]^{1/2}$$

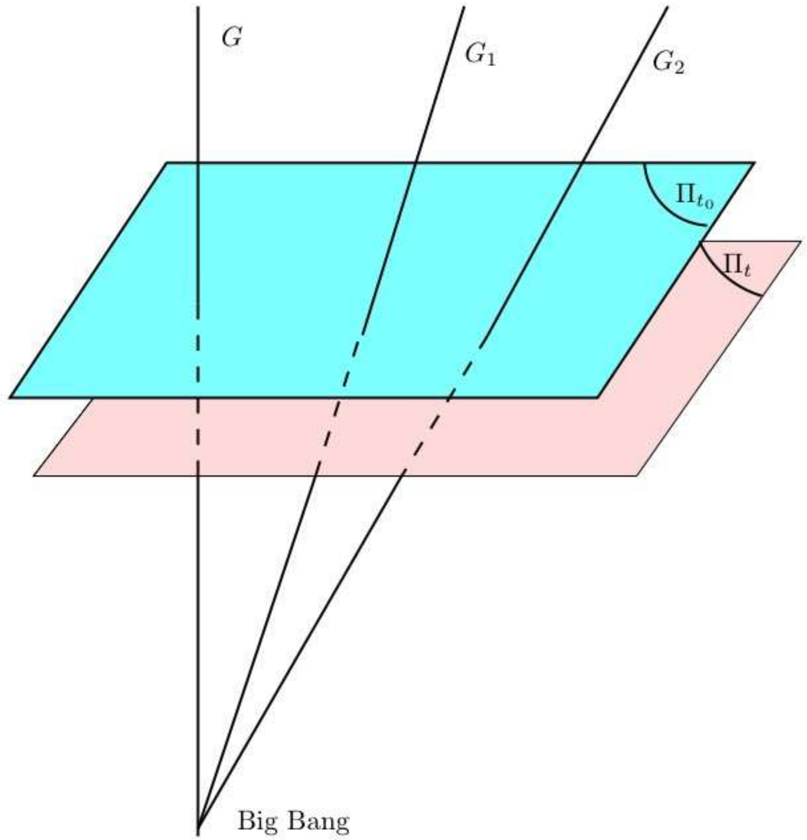

FIG. 7 – Foliation of four-dimensional space-time. Galaxies in free fall, choose time = proper time of each galaxy = time elapsed since Big Bang.

Universe expansion : $d(t)$ increases with time. Define scale factor $a(t)$ and comoving coordinates (x, y, z). Scale factor dimensionless, $a(t_0) = a_0 = 1$

$$d(t) = a(t) d_{\mathrm{com}}$$

$$X = a(t)x \qquad Y = a(t)y \qquad Z = a(t)z$$

Then galaxies have fixed coordinates ! Two galaxies separated by comoving interval $(\Delta x, \Delta y, \Delta z)$, distance at time t

$$d(t) = a(t)\left[(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2\right]^{1/2} = a(t) d_{\mathrm{com}}$$

Friedmann-Lemaître metric

$$c^2 \mathrm{d}\tau^2 = c^2 \mathrm{d}t^2 - a^2(t)\left[\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right]$$

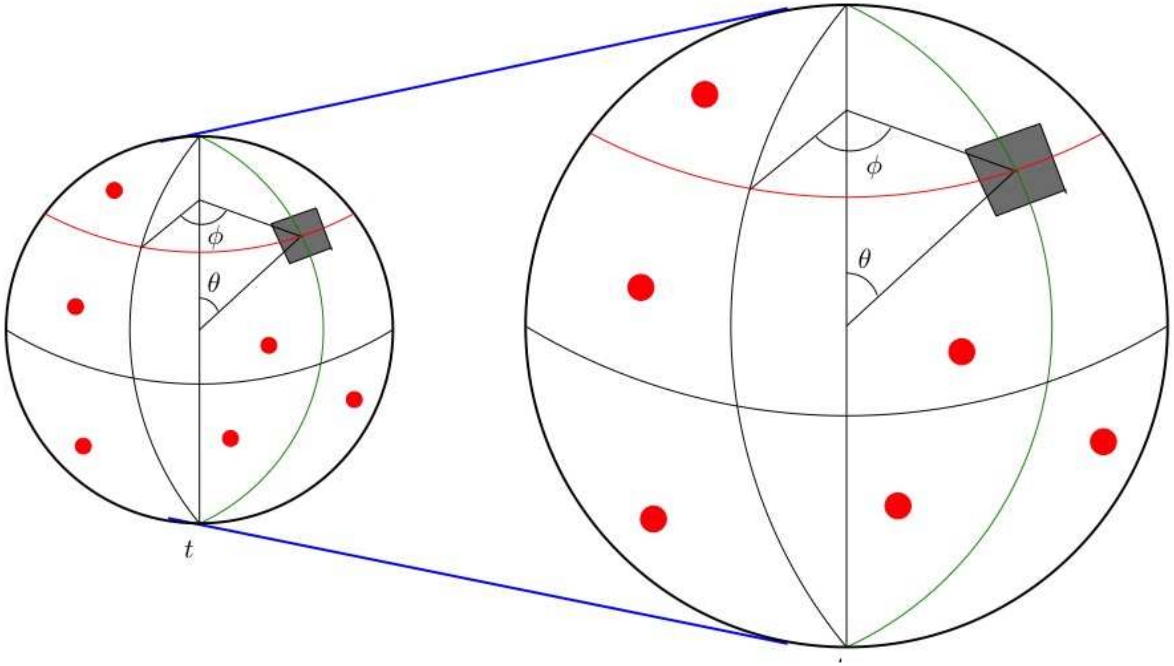

FIG. 8 – Comoving coordinates in three dimensional space-time, space with constant positive curvature.

6. Cosmological redshift

Spherical coordinates, Friedmann-Lemaître metric

$$\begin{aligned} \mathrm{d}\tau^2 &= \mathrm{d}t^2 - a^2(t)[\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2] \\ &= \mathrm{d}t^2 - a^2(t)[\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2)] \end{aligned}$$

Consider two galaxies $r = 0$ (us!) and $r = r_{\mathrm{com}}$.

Photon emission : t_e, $t_e + \Delta t_e$

Photon reception : t_0, $t_0 + \Delta t_0$

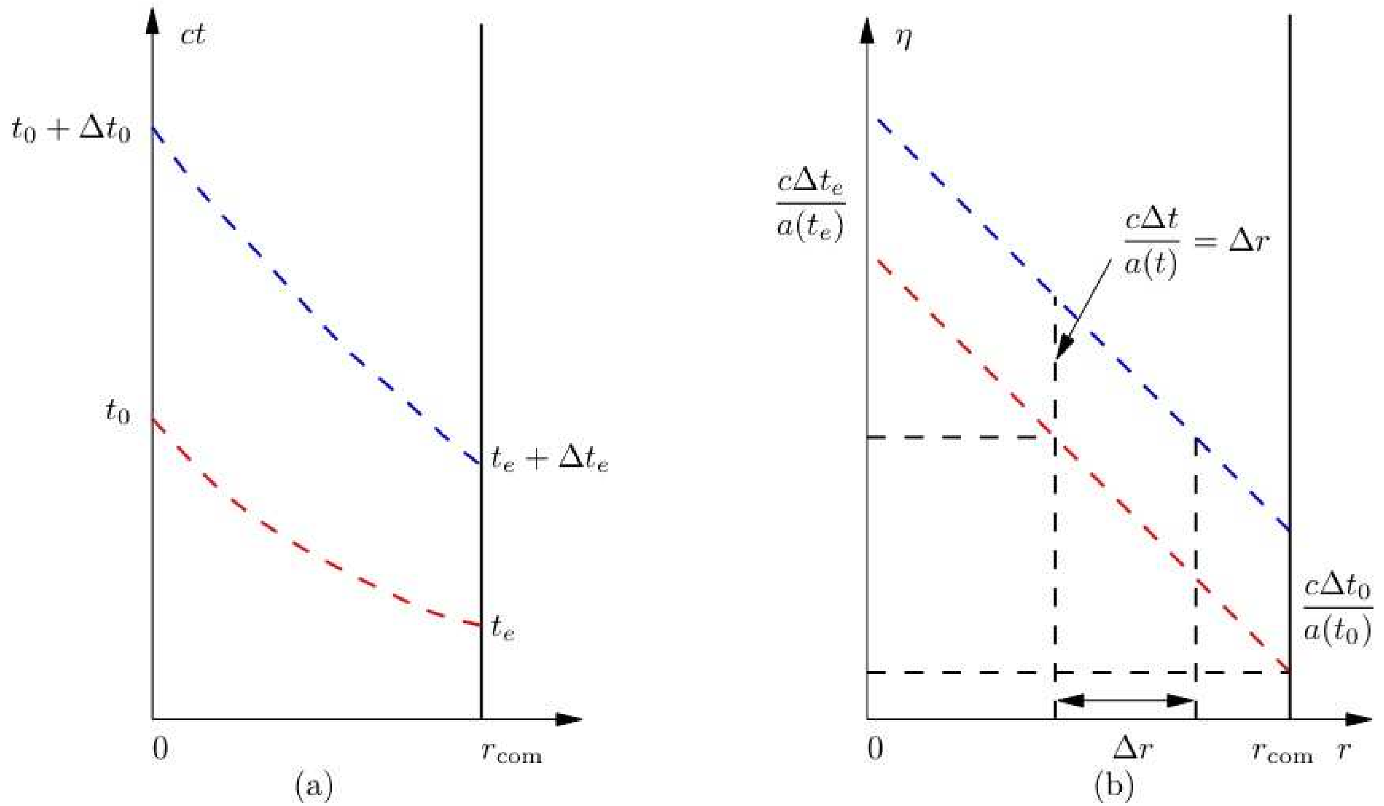

FIG. 9 – Photon propagation in (ct, r) coordinates. Propagation in conformal coordinates (η, r)

As $\mathrm{d}\tau^2 = 0$ for a photon, in comoving coordinates

$$r_{\mathrm{com}} = \int_{t_e}^{t_0} \frac{\mathrm{d}t}{a(t)} = \int_{t_e+\Delta t_e}^{t_0+\Delta t_0} \frac{\mathrm{d}t}{a(t)}$$

so that

$$\frac{\Delta t_0}{a(t_0)} = \frac{\Delta t_e}{a(t_e)}$$

cosmological redshift z

$$1 + z = \frac{\lambda_0}{\lambda_e} = \frac{\omega_e}{\omega_0} = \frac{a(t_0)}{a(t_e)} > 1$$

$z =$ observational quantity : unambiguous, model independent, while e.g. age of the universe model dependent. For example, CMB : $z = 1\,100$, farthest observable galaxies (quasars) $z \sim 10$

Conformal coordinates $t \longrightarrow \eta$

$$\eta = c \int_{t_e}^{t} \frac{\mathrm{d}t'}{a(t')}$$

Relation with Hubble's law

$$v = \dot{a}(t_0) d_{\mathrm{com}} = H_0 a(t_0) d_{\mathrm{com}}$$

so that $H_0 = \dot{a}(t_0)/a(t_0)$. Hubble parameter $H(t)$

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$

What is the meaning of $t = 0$? Big Bang instant? Classical GR, $t = 0$ is singular, curvature becomes infinite. Big Bang theory valid for $t > 0$, so the Big Bang instant is not part of the Big Bang theory!

A simple model for $a(t)$: matter dominated universe. $t_H = 1/H_0$, $t_0 < t_H$ because of deceleration

$$a(t) = (t/t_0)^{2/3}$$