Introduction to Cosmology

Lecture 3 : History of the Big Bang

Plan

1. Main features of Big Bang history
2. Phase transitions in the early universe
3. Primordial nucleosynthesis
4. Cosmic microwave background
5. Successes and shortcomings of ΛCDM
6. Inflation

1. Main features of Big Bang history

Temperature T, Boltzmann constant k_B, often $\hbar = c = k_\mathrm{B} = 1$

$$E = k_\mathrm{B} T \qquad 1\,\mathrm{eV} = 10^4\,\mathrm{K} \qquad m_p c^2 = 10^{13}\,\mathrm{K}$$

Probability of observing particle of mass m

$$P \propto \exp\left(-\frac{mc^2}{k_\mathrm{B} T}\right)$$

If $mc^2 \ll k_\mathrm{B} T$, then radiation dominated, energy density ρ

$$\rho_\mathrm{bosons} = \frac{g_b \pi^2 k_\mathrm{B}^4}{30 \hbar^3 c^3} T^4 \qquad \rho_\mathrm{fermions} = \frac{7 g_f \pi^2 k_\mathrm{B}^4}{240 \hbar^3 c^3} T^4$$

g_b, g_f = number of spin states and species

Particle interactions inhibited by expansion, characteristic reaction time or reaction rate, $\tau = 1/\Gamma$

$$\tau_{\rm char} \ll \frac{1}{H(t)} \qquad \Gamma_{\rm char} \gg H(t)$$

Otherwise particles leave thermal equilibrium.

Rate equation for particles of type i, $i + j \leftrightarrow k + l$

$$\frac{{\rm d}n_i}{{\rm d}t} = -3\frac{\dot{a}}{a}\,n_i + \Gamma_{k+l\to i+j}n_k n_l - \Gamma_{i+j\to k+l}n_i n_j$$

$\dot{a}/a = H$

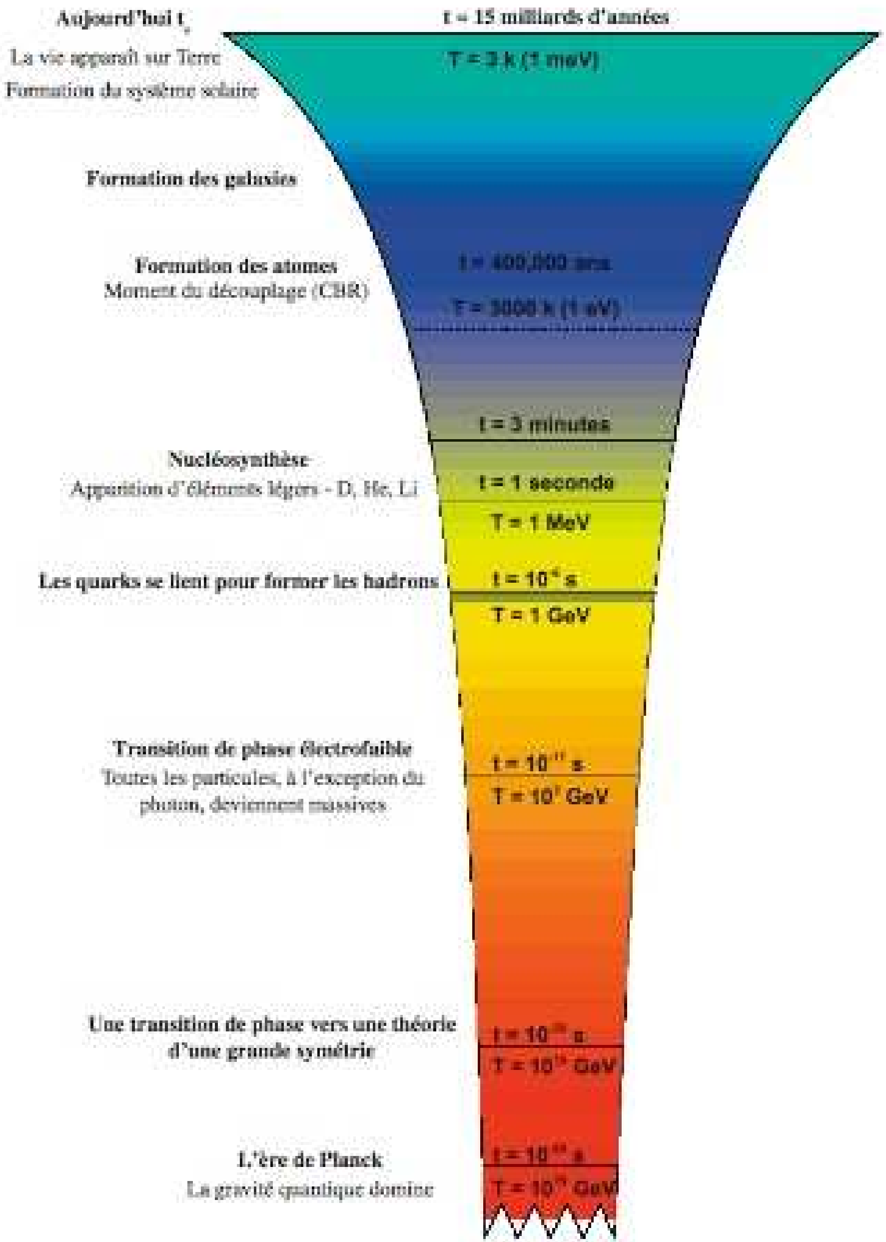

Aujourd'hui t
t = 15 milliards d'années
La vie apparaît sur Terre
T = 3 k (1 meV)
Formation du système solaire
Formation des galaxies
Formation des atomes
Moment du découplage (CBR)
t = 400,000 ans
T = 3000 k (1 eV)
t = 3 minutes
Nucléosynthèse
Apparition d'éléments légers - D, He, Li
t = 1 seconde
T = 1 MeV
Les quarks se lient pour former les hadrons
T = 1 GeV
Transition de phase électrofaible
Toutes les particules, à l'exception du photon, deviennent massives
Une transition de phase vers une théorie d'une grande symétrie
L'ère de Planck
La gravité quantique domine

Table 1: Major Events in the History of the Universe.

	Time	Energy	
Planck Epoch?	$< 10^{-43}$ s	10^{18} GeV	
String Scale?	$\gtrsim 10^{-43}$ s	$\lesssim 10^{18}$ GeV	
Grand Unification?	$\sim 10^{-36}$ s	10^{15} GeV	
Inflation?	$\gtrsim 10^{-34}$ s	$\lesssim 10^{15}$ GeV	
SUSY Breaking?	$< 10^{-10}$ s	> 1 TeV	
Baryogenesis?	$< 10^{-10}$ s	> 1 TeV	
Electroweak Unification	10^{-10} s	1 TeV	
Quark-Hadron Transition	10^{-4} s	10^{2} MeV	
Nucleon Freeze-Out	0.01 s	10 MeV	
Neutrino Decoupling	1 s	1 MeV	
BBN	3 min	0.1 MeV	
			Redshift
Matter-Radiation Equality	10^{4} yrs	1 eV	10^{4}
Recombination	10^{5} yrs	0.1 eV	1,100
Dark Ages	$10^{5} - 10^{8}$ yrs		> 25
Reionization	10^{8} yrs		$25 - 6$
Galaxy Formation	$\sim 6 \times 10^{8}$ yrs		~ 10
Dark Energy	$\sim 10^{9}$ yrs		~ 2
Solar System	8×10^{9} yrs		0.5
Albert Einstein born	14×10^{9} yrs	1 meV	0

2. Phase transitions in the early universe

10^{19} GeV, 10^{-43} s, Planck scale, quantum gravity ?

10^{15} GeV : grand unification phase transition (?). Leptoquarks acquire a mass and disappear from equilibrium

10^3 GeV : electroweak phase transition, $W^\pm$, Z^0 and Higgs acquire a mass and disappear from equilibrium

200 Mev : quark hadron phase transition. Quark-gluon plasma $\Longrightarrow$ protons and neutrons, confinement transition

1 MeV : neutrino decoupling, $T_\nu = (4/11)^{1/4}\, T_\gamma$. Plasma of charged particles : protons and electrons and neutral particles : photons, neutrons, and neutrinos

Dominance of matter over antimatter : antibaryons and positrons do not survive

$$\frac{n_B}{n_\gamma} \simeq 5 \times 10^{-10}$$

One nucleon for about 2 billions photons. No good explanation, CP violation ?

3. Primordial nucleosynthesis

Formation of light nuclei, mainly ^{4}He. Heavier nuclei up to Fe $A \leq 56$ formed in ordinary stars, still heavier nuclei $A \geq 56$ in supernovae explosions. Bottleneck at $A = 8$, no stable nucleus, need collisions of three ^{4}He nuclei : $3\,^4\text{He} \rightarrow {}^{12}\,\text{C}$

^{4}He synthesis begins $T \simeq 70\,\mathrm{keV}$ and ends $T \simeq 30\,\mathrm{keV}$. Main factor is Boltzmann factor : at equilibrium

$$\frac{n_n}{n_p}(T) = \exp\left(-\frac{(m_n - m_p)c^2}{k_{\mathrm{B}}T}\right)$$

Equilibrium governed by reactions

$$\begin{aligned} \nu_e + n &\leftrightarrow p + e^- \\ e^+ + n &\leftrightarrow p + \overline{\nu}_e \\ n &\leftrightarrow p + e^- + \overline{\nu}_e \end{aligned}$$

In practice neutron decay plays a small role. Neutrinos decouple at freezing temperature T_F, $H(T_F) \sim \Gamma$

$$\Gamma(T) \simeq n_\nu(T)\langle\sigma v\rangle \sim G_F^2 T^5$$

($G_F =$ Fermi constant). Freezing temperature T_F given by

$$G_F^2 T_F^5 = \left(\frac{8\pi}{3} G\rho\right)^{1/2} \qquad \rho \propto g T^4$$

$g =$number of states, spin and species. Numerically

$$T_F \sim g^{1/6} \times \text{MeV}$$

Allows us to compute the n_n/n_p ratio at T_F, $n_n/n_p(T_F) \simeq 1/6$

When T drops to about $\varepsilon_B/25$, $\varepsilon_B =$ deuterium binding energy $= 2.23$ MeV, almost all neutrons are bound in helium nuclei : ^{4}He formation ends at about $t = 3$ minutes. Also small amount of deuterium, lithium and ^{3}He.

Why 70 keV ? $n + p \leftrightarrow d + \gamma$

$$n_p n_n \langle \sigma_{np} v \rangle_T = n_d n_\gamma \langle \sigma_{d\gamma} \rangle_T$$

but $n_p \ll n_\gamma$ so that n_d stays small until $E_\gamma \ll 2.23\,\text{MeV}$

4. Cosmic Microwave Background

For $T \gg 3000\,\mathrm{K}$, mean free path of photons very short due to Thomson scattering on free electrons

$$\gamma + e^- \leftrightarrow \gamma + e^-$$

universe opaque to photons.

$T \sim 3000\,\mathrm{K}$, $t \simeq 380\,000$ years : atoms begin to form, mean free path increases dramatically and universe becomes transparent to photons, photons decouple. Surface of last scattering, decoupling takes about 50 000 years. Why 0.3 eV rather than ionization energy 10 eV ?

After decoupling, gas of free photons whose dynamics is governed uniquely by expansion, $\lambda \propto a(t) \propto 1/T$. Today CMB temperature $= 2.73\,\mathrm{K}$, so $z_{\rm dec} \simeq 1\,100$

CMB temperature fluctuations

Satellites : COBE (1992), WMAP (2003), Planck (2014)

Gives a snapshot of the universe 380 000 years after Big Bang

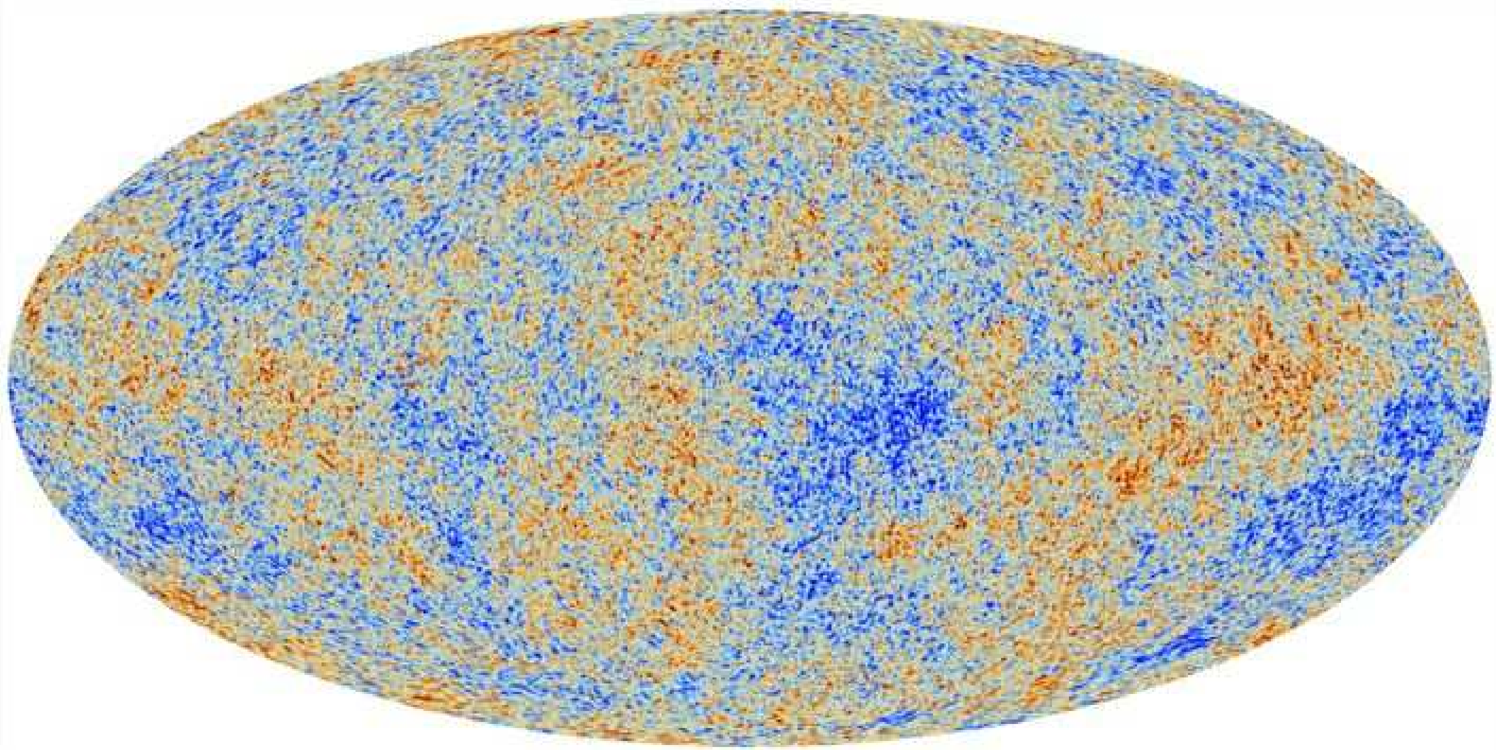


FIG. 3 – The Cosmic Microwave Background as measured by Planck. Orange (blue) zones : higher (lower) temperatures. universe 13.4 billion years ago

Fluctuations on the order of 10^{-5}, very important, believed to be the seed for the formation of large structures at $z \sim 10$. Temperature fluctuations reveal density fluctuations.

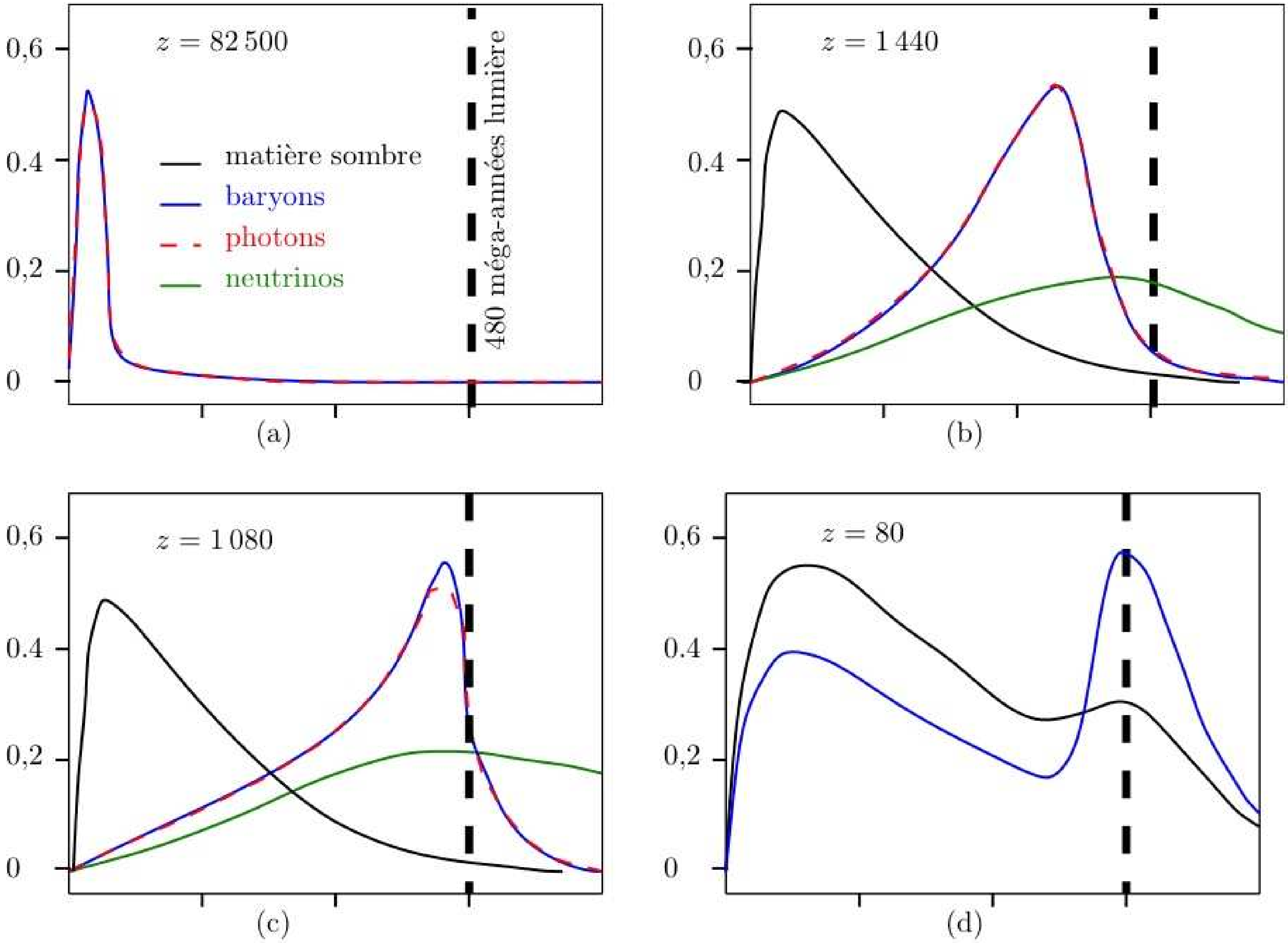
Large structure formation : gravitational instability. Two effects work against it

1. universe expansion, tends to dilute matter (Lifshitz and Zeldovich)
2. Radiation pressure, as in stars

Understanding temperature fluctuations. Sound waves in the plasma : photons, baryons, electrons and neutrinos (decoupled), dark matter with gravitational coupling only. Sound waves with velocity $c_s = c/\sqrt{3}$, related to $\mathcal{P} = \rho/3$.

How a sound wave develops

1. $z = 82\,000$ ($t \simeq 110$ years). A fluctuation creates an excess of density at the origin
2. $z = 1\,440$ ($t \simeq 0.2 \times 10^6$ years). Protons and photons are strongly coupled and propagate to the right
3. $z = 1\,080$ ($t \simeq 0.38 \times 10^6$ years). Protons and photons decouple and sound wave stops
4. $z = 80$ ($t \simeq 23.4 \times 10^6$ years). Because of gravitation, dark matter distribution becomes roughly similar to proton distribution. Acoustic peak reflected in distribution of galaxies


$z = 82\,500$
0,6
0.4
0,2
0
matière sombre
baryons
photons
neutrinos
480 méga-années lumière
(a)
$z = 1\,440$
(b)
$z = 1\,080$
(c)
$z = 80$
(d)

First consequence of acoustic peak : distribution of galaxies. Distance traveled by the peak in approximate model (see lecture 2), or acoustic horizon

$$l = ct_0[3/(1 + z_{\text{dec.}})]^{1/2}$$

$\sqrt{3}$ comes from sound velocity. With $t_0 \simeq 1.4 \times 10^{10}$ years we find $l \simeq 7.3 \times 10^8$ l-y, numerical value $l \simeq 4.8 \times 10^8$ l-y

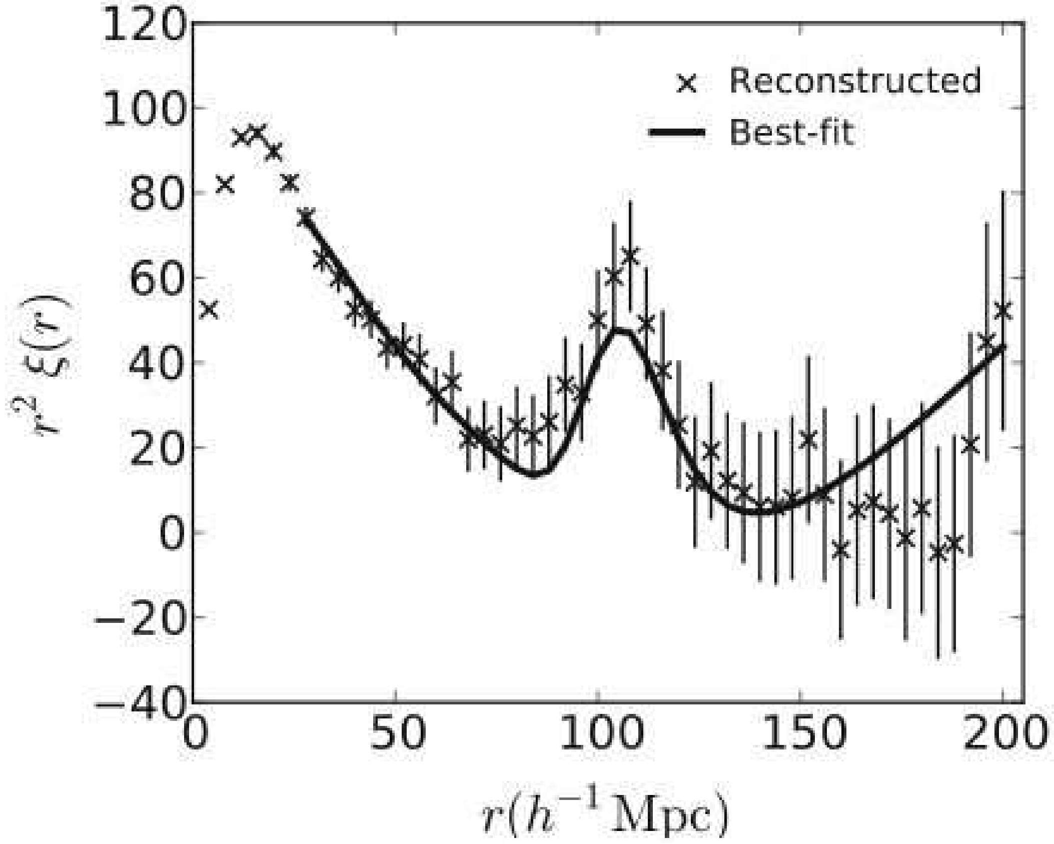


FIG. 5 – Footprint of the acoustic peak in the galaxy distribution. Vertical axis : distance between galaxies for all possible pairings. Horizontal axis : comoving distance, $h = 0.71$. The peak in the distribution is located at 480 000 l-y.

Angular analysis of temperature fluctuations

$$\Delta t(\hat{n}) = \sum_{l,m} a_{lm} Y^{lm}(\theta, \varphi) \qquad \hat{n} = (\theta, \varphi)$$

Harmonic analysis (peak structure) gives information on : baryonic matter density, dark matter density, dark energy density, space curvature

Characteristic angular scale of 1 degree

$$\theta \simeq [3(1 + z_{\text{dec.}})]^{-1/2}$$

Peak at $l \simeq \pi/\theta \simeq 200$

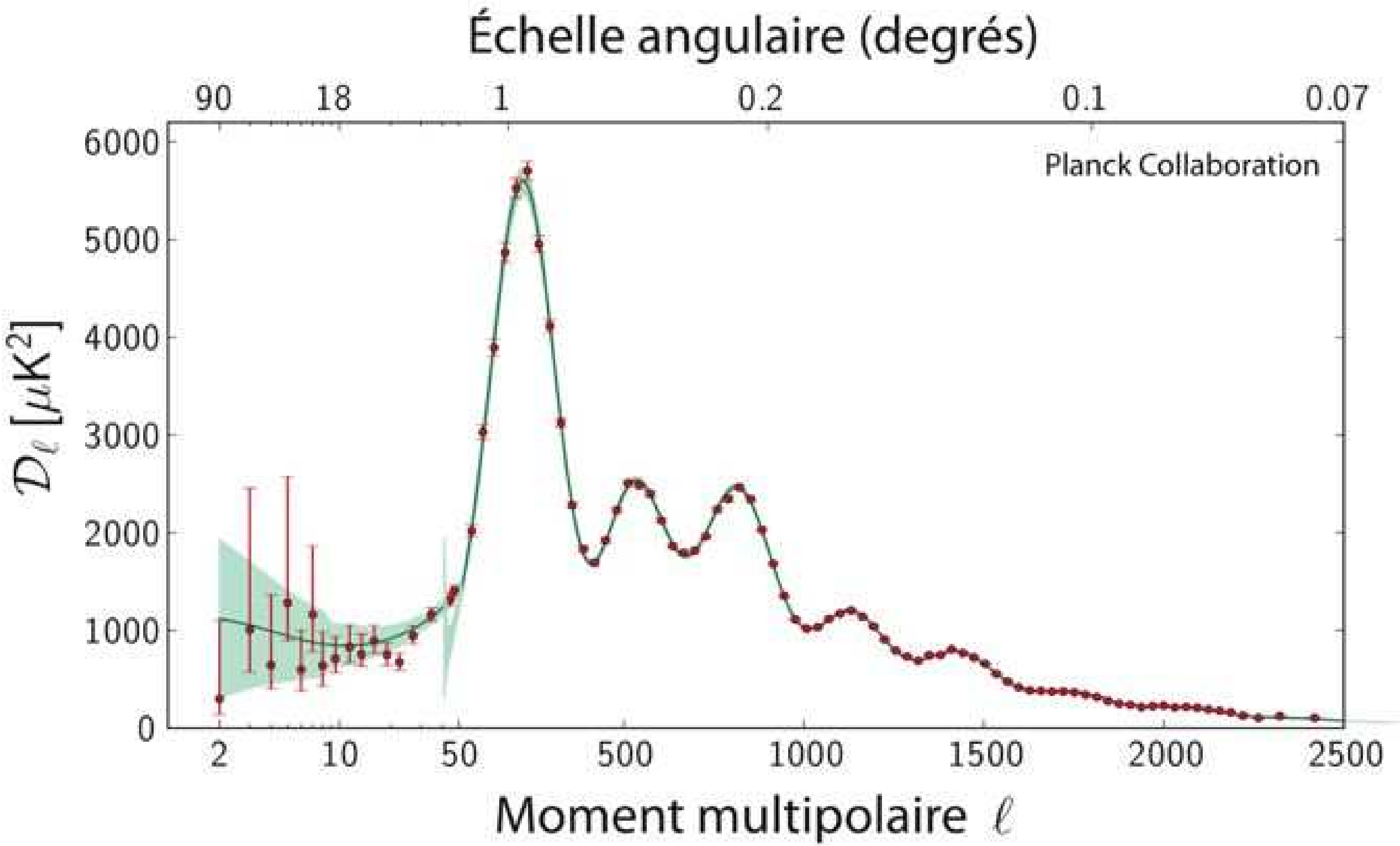


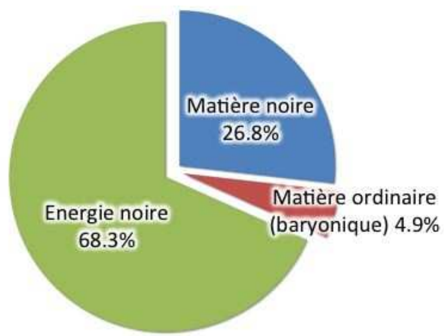
FIG. 6 – Angular analysis of temperature fluctuations as measured by Planck

5. Successes and shortcomings of ΛCDM

Standard model of cosmology : ΛCDM. Λ= cosmological constant, CDM = Cold Dark Matter. Cold = non relativistic particles.

All observations consistent with the following proportions

1. Ordinary, or baryonic, matter : 5%
2. Dark matter : 25%
3. Dark energy : 70%



Successes of ΛCDM

1. Consistent description of the history of the universe
2. Correct prediction of the amount of ^{4}He
3. Correct prediction of CMB and of its temperature
4. Explanation of angular scale of temperature fluctuations

Shortcomings of ΛCDM

1. Origin of dark matter and dark energy unknown
2. Horizon problem
3. Flatness problem and fine tuning

Dark matter : stable particles, search for WIMPS (weakly interacting massive particles), maybe LSP : neutralinos (?). Modified gravity at very small accelerations (MOND) ?

Flatness problem

$$\Omega(t) - 1 = \frac{kc^2}{R^2\dot{a}^2(t)} \qquad \Omega(t) = \frac{8\pi G\rho(t)}{3H^2(t)}$$

Matter dominated universe

$$\Omega(t) - 1 = (\Omega_0 - 1)\frac{a(t)}{a(t_0}$$

$\Omega = 1$ unstable solution ! If $\Omega_0 - 1 \simeq 10^{-2}$, then at ^{4}He formation $\Omega(t) - 1 \simeq 10^{-16}$! Needs fine tuning, considered as not natural. Solution(?) to 2 and 3 : inflation

6. Inflation

Conformal time τ $(H(a) = \dot{a}/a)$

$$\tau = \int^t \frac{\mathrm{d}t}{a(t)} = \int^a \frac{\mathrm{d}a}{a}\frac{\mathrm{d}t}{\mathrm{d}a} = \int^a \frac{\mathrm{d}a}{a^2 H(a)}$$

Radiation dominated : $H(a) = H_0(a_0/a)^2$

$$\tau = \frac{1}{a_0 H_0}\, a$$

Matter dominated : $H(a) = H_0(a_0/a)^{3/2}$

$$\tau = \frac{2}{H_0 a_0^{3/2}}\, a^{3/2}$$

In both cases τ increases with a. The horizon problem comes from decelerating universe : $\ddot{a} < 0$. Look for $\ddot{a} > 0\,!$

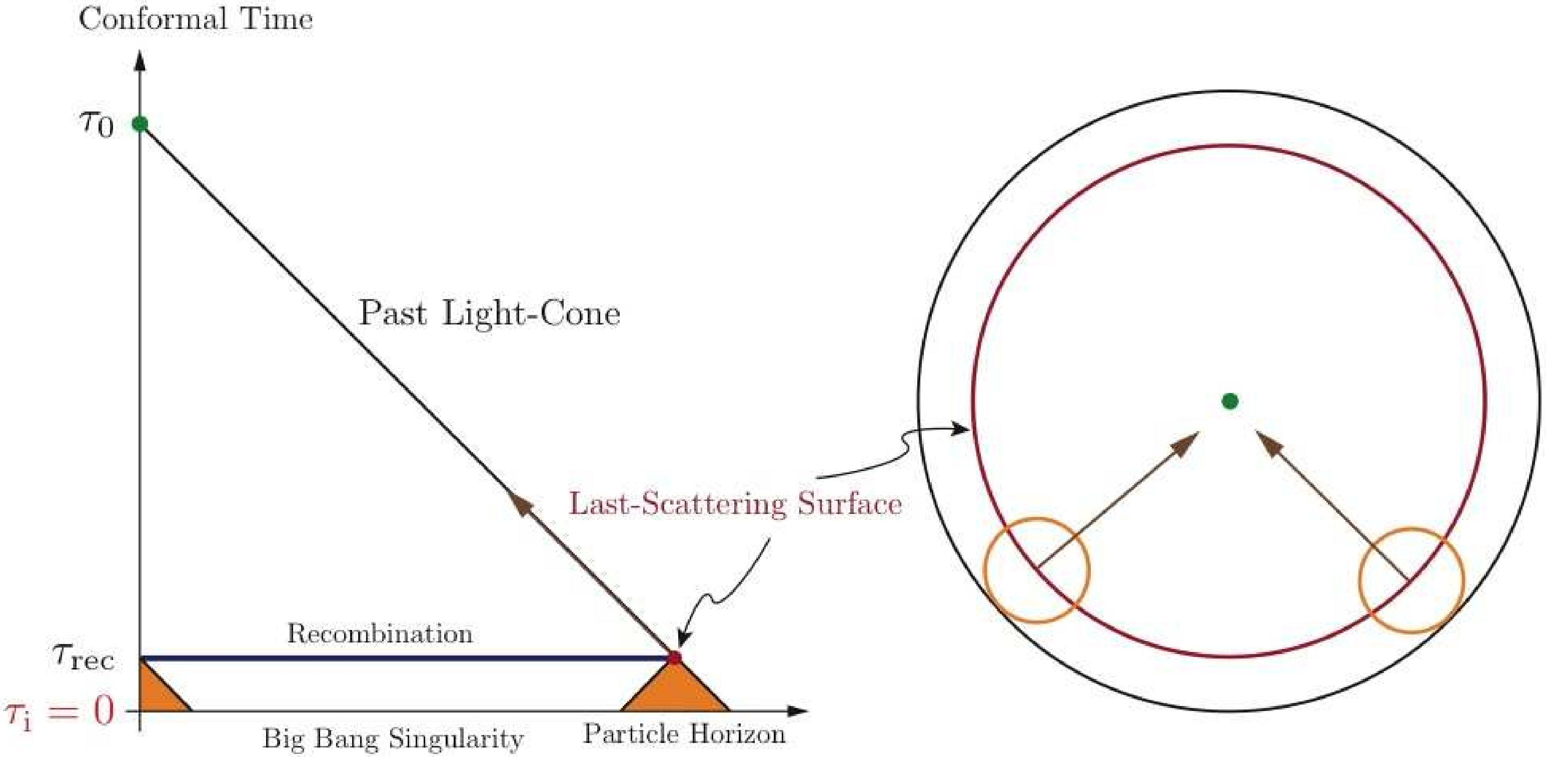


FIG. 7 – Conformal diagram of standard Big Bang

Inflation phase : $a_2 \leq a \leq a_1, \quad a_2 \ll a_1$. Inflation begins at a_2 and ends at a_1. Assume $\rho(t) = \rho_1 =$ constant in interval

$$\frac{\dot{a}}{a} = \pm\sqrt{\frac{8\pi G}{3}\rho_1} = \pm\sqrt{H_1} = \mathrm{cst}$$

$$\tau(a) = -\int_a^{a_1} \frac{\mathrm{d}a}{a^2 H_1} = \frac{1}{H_1}\left(\frac{1}{a_1} - \frac{1}{a}\right) \simeq -\frac{1}{H_1 a}$$

$\tau \rightarrow -\infty$ if $a \rightarrow 0$: the conformal time increases when a decreases ! and the $a \rightarrow 0$ singularity is pushed to $\tau \rightarrow -\infty$. Note $\tau \propto 1/a$ and not $a^\alpha, \ \alpha > 0$. Distance to horizon

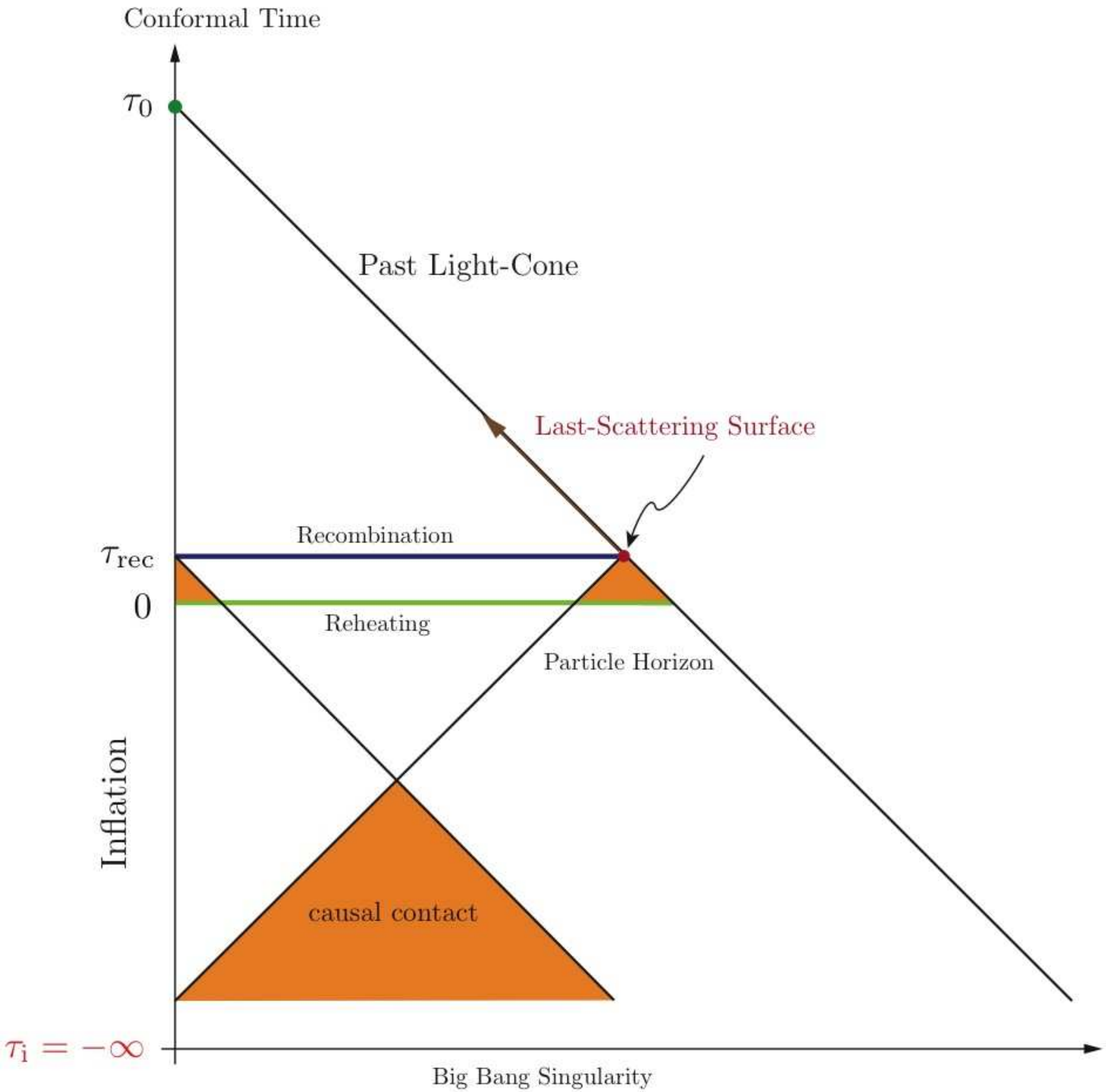
$$d_{\mathrm{hor}}(a) = a_0 \int_{a_2}^{a} \frac{\mathrm{d}a}{a^2 H_1} \sim \frac{a_0}{H_1 a_2} = \mathrm{constant}$$

Main contribution to d_{hor} comes from $a \rightarrow 0$!

Condition for inflation : comoving Hubble radius $(aH)^{-1}$ decreases with time

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{aH}\right) < 0 \Longrightarrow \frac{\mathrm{d}^2 a}{\mathrm{d}t^2} > 0 \Longrightarrow (\rho + 3\mathcal{P}) < 0$$

The comoving Hubble radius $1/(aH$ decreases.


Conformal Time
τ_0
Past Light-Cone
Last-Scattering Surface
Recombination
τ_{rec}
0
Reheating
Particle Horizon
Inflation
causal contact
$\tau_i = -\infty$
Big Bang Singularity

Solution of the horizon problem : horizon should include present horizon

$$d_{\rm hor}(a_1) \geq H_0^{-1} \Longrightarrow \frac{a_0}{a_1}\,\frac{a_1}{a_2} H_1^{-1} \geq H_0^{-1} \Longrightarrow \frac{a_1}{a_2} \sim \frac{a_1}{a_0}\sqrt{\frac{\rho_1}{\rho_0}}$$

Compute H_1 : radiation dominated at a_1 (end of inflation)

$$\frac{\rho_1}{\rho_0} = \frac{\rho_R(t_0)}{\rho_M(t_0)}\left(\frac{a_0}{a_1}\right)^4 \sim 10^{-4}\left(\frac{a_0}{a_1}\right)^4$$

Therefore

$$\frac{a_1}{a_2} > 10^{-2}\,\frac{a_0}{a_1}$$

If inflation corresponds to GUT phase transition, $T \sim 10^{15}$ GeV, $a_0/a_1 \sim 10^{28}$

$$\frac{a_1}{a_2} \sim 10^{26} \sim \mathrm{e}^{60}$$

Duration : $60/H_1 \sim 10^{-34}\,\mathrm{s}$

Flatness problem

$$|1-\Omega(a)| = \frac{1}{(aH)^2}$$

$(aH)^{-1}$ decreases during inflation, drives universe toward flatness.

Model for inflation : scalar field (inflaton) $\phi(x^\mu) \simeq \phi(t)$ coupled to gravity

$$S = \int \mathrm{d}^4x\sqrt{g}\left[\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

From $T^{\mu\nu}$ obtain ρ and $\mathcal{P}$

$$\begin{aligned}\rho_\phi &= \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ \mathcal{P}_\phi &= \frac{1}{2}\dot{\phi}^2 - V(\phi)\end{aligned}$$

If $V \simeq$ constant and $\dot{\phi} \simeq 0$, then $\rho + 3\mathcal{P} = -2V(\phi) < 0$

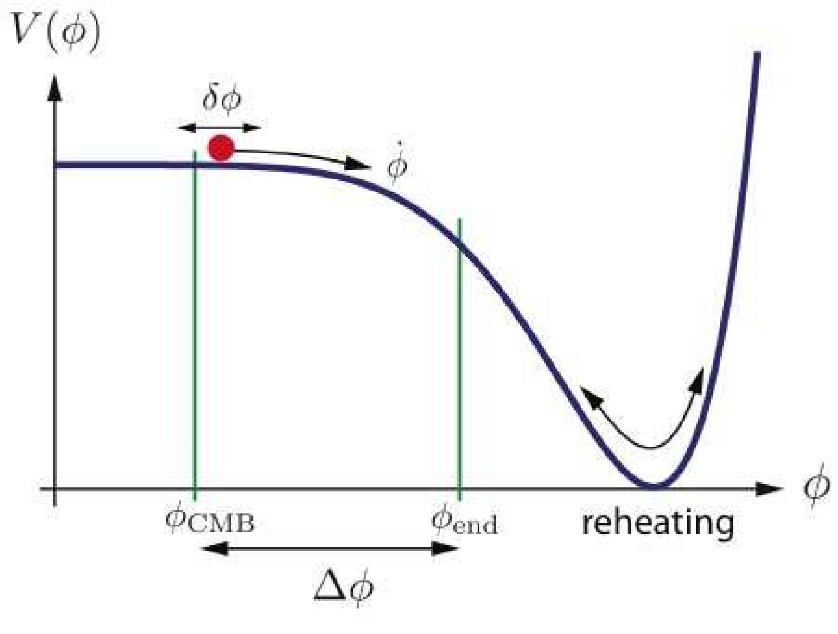


FIG. 9 – The inflaton potential $V(\phi)$

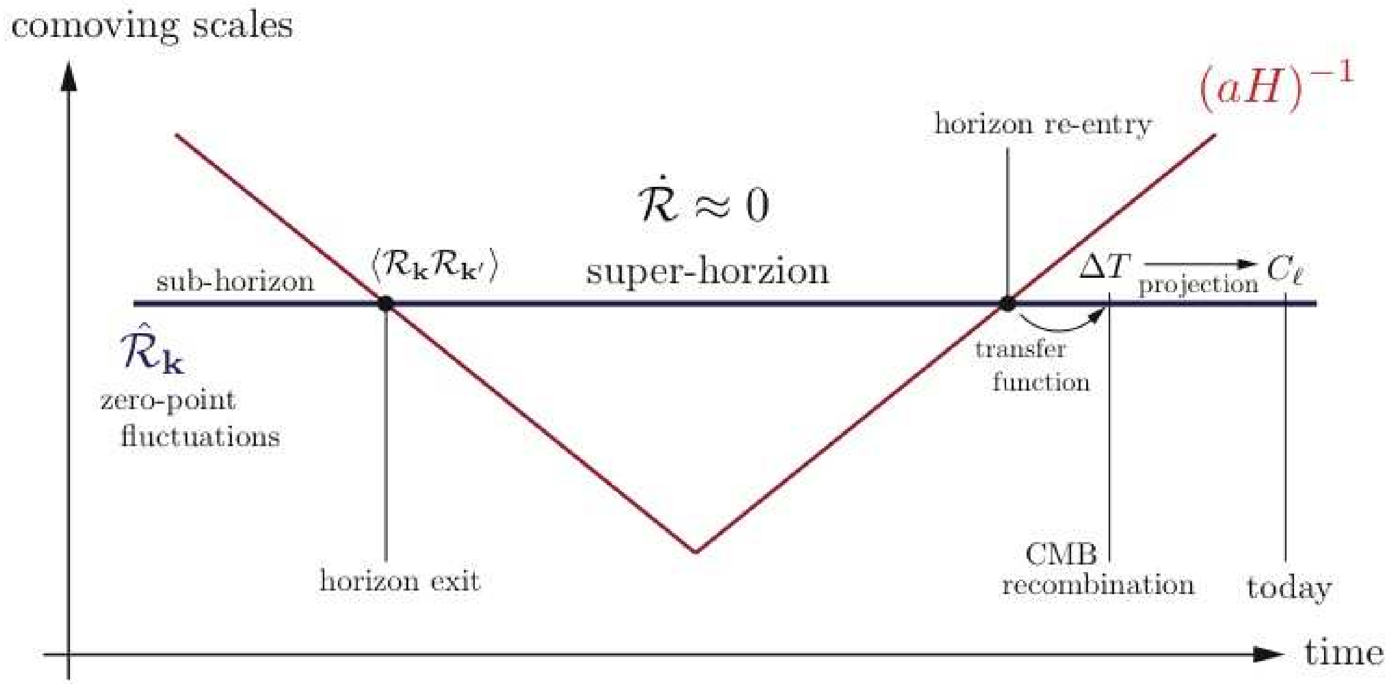


FIG. 10 – Quantum fluctuation of the inflaton field leads to fluctuations observed in CMB and galaxy distribution

Two divergent points of view on inflation

1. Inflation is part of the standard model of cosmology
2. Inflation raises more problems than it solves